Resit 3 / Numerical Mathematics 1 / July 6th 2021, University of Groningen

Instructions

- This examination is *open book*, that is you are allowed to check any material of the course.

- **You have 3h20m to upload your solutions. People with special needs (according to the official information of the Educational support center) have additional 30 minutes.**

- Upload the answers on the same location in Nestor where you downloaded them, not in your File Exchange.

- The grade will be computed as the number of obtained points, plus 1.

- Do not communicate with other students during the examination.

- If you did not write Test 1 and hence you have not yet upload the pledge, please sign the pledge and upload it as a separate file in the same place where you will upload your answers.

- Stay in the online main collaborate room during the examination. Information delivered there is official and part of the instructions of the examination. The activity in the room will be recorded.

- The examination must be written by hand **in a tidy and legible way**, scanned and uploaded as PDF to Nestor. Of course you can also use a tablet to write your answers.

- **Upload the PDF in vertical orientation, such that it requires no rotation to be readable.**

- All answers need to be justified using mathematical arguments.

- Oral checks may be run afterwards, either randomly and/or in case of suspicion of fraud.

- **If you do not follow these instructions you will receive the minimal grade.**

Questions

Consider the system of ODEs for the functions $y(t), x(t)$:

$$y' = -\alpha y^p + x^q y^m \ , y(0) = y_0 \neq 0$$
$$x' = -x^{q-1} y^{m+1} - \gamma x^r \ , x(0) = x_0 \neq 0 \tag{1}$$

with $\alpha, \gamma > 0$ and $p, q, m, r \in \mathbb{N}$.

(a) $\boxed{4.5}$ For $q = 1, m = 0, r = 1, p = 1$, apply the β-method to Problem (1). Assuming then $\alpha = \gamma$, find the values of $h_{crit}(\beta)$ such that, if the time step $h < h_{crit}(\beta)$ for the whole range $0 \leq \beta \leq 1$ then the discrete solution is asymptotically stable.

(b) $\boxed{4.5}$ Propose a new discretization for Problem (1) – detailing all terms – such that the resulting discrete problem is linear for the unknowns x_{n+1}, y_{n+1} but it still holds that $x_{n+1}^2 + y_{n+1}^2 < x_n^2 + y_n^2$ for p, r odd numbers (prove it). *Hint*: you could extend the method of Question (b) in Test 3 for Problem (1).